C_L = impurity concentration (molar ratio) in the mother liquor
C_o = equilibrium impurity concentration (molar ratio) in the crystal
C_s = impurity concentration (molar ratio) on the crystal surface
C_s^o = equilibrium impurity concentration (molar ratio) on the surface
D_i = diffusion coefficient of component i in the crystal
h = thickness of a single layer
k_{eff} = effective distribution coefficient of an impurity (crystal-to-solution)
R = linear growth rate of a crystal face

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Manuscript received September 1, 1966; revision received November 8, 1966; paper accepted November 8, 1966. Paper presented at AIChE Houston meeting.

Process Control by Digital Compensation

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Discrete control algorithms, suitable for programming in a direct digital control computer, are presented. For processes whose dynamics can be adequately modeled as first order with delay, digital compensation algorithms are derived to yield theoretically a response with finite settling time, when the system is step forced in either set point or load. The utility of the proposed designs is experimentally verified by application to a higher order physical process whose dynamics are not fully described by the model. The results demonstrate that sampling frequencies may be reduced considerably below presently accepted values while still maintaining transient response characteristics of the system comparable with those obtainable from conventional continuous control.

The small digital computers now available for process control have demonstrated their ability to replace adequately conventional analog control systems. In designing new plants and modernizing existing facilities, the process industry is giving more consideration to the possibility of direct digital control (DDC) as an alternative to conventional pneumatic or electronic techniques. Williams (*24, 25*) reports that computers have been reduced to one-quarter of their original cost, while reliability has increased tenfold. In addition, computer capability has greatly expanded.

Several workers in the field have reported the use of discrete versions of conventional control algorithms (*4, 6, 17, 23*). The performance of these systems under computer control is of course limited to that which is obtainable from their continuous-data analogs. In these studies sampling is performed as often as 1/sec., which is essentially continuous control for most chemical processes.

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In 1954 Bergen and Ragazzini (2) introduced the technique of digital compensation, which is applicable to systems where both the error and manipulated variable are being sampled. The concept of minimal prototype design is presented. In this design a digital controller operating in a feedback configuration drives the output of the system to zero steady state error at sampling instants in a minimum number of sampling periods, for a given input (for example, step, ramp, etc., in set point). Such a system is said to have a finite settling time equal to this minimum number of sampling periods. Little or no consideration is given to the state of the system between sampling instants nor to the response of the system to an input other than the one for which it is originally designed.

Since 1954 many articles have appeared which extend the theory to include a variety of problems (*8, 9, 16, 21, 22*). All standard sampled-data textbooks contain a detailed description of the digital compensation design, but despite the considerable theoretical interest in this technique, the actual application of digital compensation to physical systems has been neglected. Even reports on simulation of digital compensation are scarce. One reasonable explanation for this neglect is that, whereas the previous studies of digital compensation consider only set-point control, the process industry is more often interested in regulator action for DDC. Furthermore, most control engineers are probably not familiar with this approach. Hence, the purpose of this paper is threefold: to extend the digital compensation theory to include the regulator problem for unmeasured, deterministic loads with nonzero mean; to present compensation algorithms which are suitable for application to physical systems where modeling error and noise inevitably exist; and to verify these designs on a real process. Designs will be presented for both steps in set point and load for the chosen model.

We restrict attention to the class of processes whose dynamics can be adequately modeled by a first-order transfer function with pure delay.

$$G_p(s) = K_p \frac{e^{-a\tau s}}{\tau s + 1} \tag{1}$$

Adequate modeling is interpreted here to mean that a controller designed to operate on Equation (1) provides acceptable control of the actual process. This model was originally proposed by Ziegler and Nichols (*26*) and has been shown by other investigators (*3, 7, 11, 13, 15*) to represent adequately the dynamics of a large class of processes. The proposed designs will be experimentally verified on a higher order process whose dynamics are not fully represented by the model.

OPERATING CHARACTERISTICS OF COMPUTER CONTROL

A DDC computer is used to control a number of process loops on a time-shared basis. We shall consider a typical loop, and other loops in the system may be treated in a similar manner. At the end of the sampling period for this particular loop, the computer samples the output of the loop and compares it with the desired value (that is, set point) to form a value for the error. The computer then calculates a new value for the manipulated variable. The manipulated variable for this loop is then held constant at the value calculated by the computer until the loop is again sampled. The computer memory is used to store sequentially past values of the error and manipulated variable. Only a small number of the most recent values, as defined by the algorithm, are retained in the computer.

The control algorithm utilizes a linear combination of the past history of the system in forming a new value for the manipulated variable. The absolute position $m(t)$ of the final control element is determined from the formula

$$m[nT] = \sum_{i=0}^{k} g_i\, e[(n-i)T] - \sum_{j=1}^{p} h_j\, m[(n-j)T] \tag{2}$$

Equation (2) gives the value at which $m(t)$ is to be held constant during the entire $(n+1)^{st}$ sampling period, that is, $m(t) = m(nT)$ for $nT \leq t < (n+1)T$. T is the sampling period and the g's and h's in Equation (2) are all constant. In this algorithm only the $k+1$ most recent values of the error and the p most recent values of the manipulated variable need be stored. The design objective is to determine suitable values of T, $\{g_i\}$, and $\{h_j\}$.

Designs will be presented in this paper for both regulator and servo (set-point) digital compensation algorithms. The computer will normally be functioning as a regulator. However, at infrequent intervals there will be a request for a change in set point for one of several reasons. For example, in a multilevel computer system, the supervisory optimizing computer may call for new set points to achieve optimum processing conditions. The computer may still use the regulator algorithm, since our design will insist that stable control be provided for step changes in set point. However, a better choice is to use a digital compensation algorithm specifically designed for step changes in set point. All servo algorithms are also of the same form as Equation (2), and differ from the regulator algorithms only in that a different set of constants is used for the calculation of the manipulated variable. For the next several sampling periods after the requested change in set point, the computer will continue to use the servo algorithm to allow the process output to settle at the new set point, after which time the loop is returned to regulator control. Although the servo algorithm is designed specifically for step changes in set point, our algorithm will also be designed to compensate adequately for unmeasured load disturbances which might enter the system while the loop is under servo control.

PERFORMANCE SPECIFICATIONS

The minimal prototype design of Bergen and Ragazzini (2) considers the response of the system only at instants of sampling. The requirements of a minimal prototype system are:

1. The compensation algorithm must be physically realizable.
2. The output of the system must have zero steady state error at sampling instants.
3. The output should equal the input in a minimum number of sampling periods.

However, for application of digital compensation to real systems, several additional constraints should be included.

4. The digital compensation algorithm should be open-loop stable.
5. Unstable or nearly unstable pole-zero cancellations should be avoided, since exact cancellation in real processes is impossible, and the resulting closed-loop system may be unstable or excessively oscillatory.
6. The design should consider the entire response of the system to eliminate hidden oscillations (intersample ripple).
7. In addition to the system responding optimally to a given test input, it should perform satisfactorily for other possible inputs and disturbances.

These additional constraints are necessary to ensure that the proposed compensation algorithms will perform satisfactorily on real systems. To meet these requirements, the resulting system may respond with a settling time longer than the minimal prototype settling time. However, the concept of finite settling time is used only as a theoretical performance criterion. In real systems, where modeling error, noise, and momentary disturbances are present, it is not possible to bring the state of the system completely to rest. This does not negate the value of the theoretical concept of finite settling time, because systems designed to meet this requirement theoretically will be shown to give satisfactory performance in real tests. In addition, all digital compensation algorithms to be presented contain the equivalent of an integrator which ensures zero steady state offset for step disturbances or inputs, regardless of modeling error and/or where the disturbance enters the loop. Finally, the minimal prototype concept provides the basis for a systematic approach to the design of digital compensation algorithms.

The next section deals with the analysis and design of digital compensation systems. Z transformation is used to analyze these sampled-data systems. The state-variable approach may also be used to design these same digital compensation algorithms. However, no real computational advantage is gained by using state-variable techniques on single-manipulated-input single-output processes. The Z transform approach does offer computational advantages for treating pure delay time, and this is the primary motivation for its application.

ANALYSIS AND DESIGN OF DIGITAL COMPENSATION SYSTEMS

If the amount of time required by the computer to sample any given loop and calculate a new manipulated variable is assumed to be negligible with respect to the time constants of the process, the operation of the digital computer may be mathematically represented by placing an ideal sampler at the computer input and an ideal sampler and zero-order hold at the computer output as illustrated in the block diagram of Figure 1*a*. An example of the input-output relationship of the ideal sampler and zero-order hold is shown in Figure 1*b*. The zero-order hold has the transfer function

$$H(s) = \frac{1}{s}\,[1 - e^{-Ts}] \tag{3}$$

Unmeasured load disturbances are assumed to enter the system at the same point as the manipulated variable and at an instant of sampling. Hence, the transfer function relating the process output to a load disturbance, and the transfer function relating the process output to the manipulated variable, are both $G_p(s)$. For loads entering at other points in the loop and/or between sampling instants, the same analytical techniques may be applied by using the appropriate $U(s)$. Thus, no loss in generality occurs.

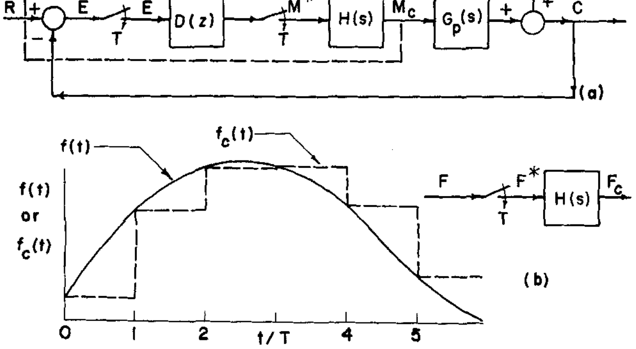

Fig. 1. *(a)* Block diagram of computer control system. *(b)* Example of input-output relationship of a sample-and-hold device.

The relationship among output, set point, and load, illustrated in Figure 1*a*, can be written in Z transform notation as

$$C(z) = \frac{HG_p(z)D(z)R(z)}{1 + HG_p(z)D(z)} + \frac{G_pU(z)}{1 + HG_p(z)D(z)} \tag{4}$$

The design technique is to specify first a desired, physically realizable output $C_d(z)$, at sampling instants, for a specific input (set point) or disturbance (load). The required digital compensation may then be determined from Equation (4). For the servo problem $[G_pU(z) = 0]$

$$D(z) = \frac{C_d(z)}{HG_p(z)\,[R(z) - C_d(z)]} \tag{5}$$

For the regulator problem $[R(z) = 0]$

$$D(z) = \frac{G_pU(z) - C_d(z)}{HG_p(z)\,C_d(z)} \tag{6}$$

The design for each of these problems is considered separately, since the required compensation for each is different.

The design of the required digital compensation will depend only on the response of the system at instants of sampling. Therefore, it will be necessary to verify that the system does not have unsatisfactory intersample behavior. There are at least two ways to check for hidden oscillations (that is, intersample ripple). One technique is to analyze the system by the modified Z transform after the compensation $D(z)$ has been designed. The entire response can then be verified to have finite settling time.

A second and somewhat easier approach is to find the corresponding response of the manipulated variable $M(z)$. For linear, time-invariant, overdamped processes, if the response of the closed-loop system has zero steady state error at sampling instants, and if the manipulated variable also has a finite settling time, then we are assured that no hidden oscillations exist because the system is receiving a constant input. It is not necessary to use the modified Z transform on the manipulated variable, because it is a piecewise constant signal and its values at sampling instants completely describe its response. If the same system responds with finite settling time, but the manipulated variable continues to oscillate, the response of the system must obviously have an intersample ripple. From the block diagram of Figure 1*a*

$$M(z) = \frac{D(z)R(z)}{1 + HG_p(z)D(z)} - \frac{D(z)G_pU(z)}{1 + HG_p(z)D(z)} \tag{7}$$

Combination of Equations (4) and (7) for the servo problem $[G_pU(z) = 0]$ implies

$$M(z) = \frac{C(z)}{HG_p(z)} \tag{8}$$

and for the regulator problem $[R(z) = 0]$

$$M(z) = -D(z)C(z) \tag{9}$$

The design equations, Equations (5) and (6), require the Z transforms $HG_p(z)$, $G_pU(z)$, $R(z)$, and the desired output $C_d(z)$. The Z transforms of the process, disturbance, and input dynamics will be presented first. Then the output transforms $C_d(z)$ which meet the design requirements will be derived.

From the chosen model, Equation (1), and the definition of the zero-order hold, Equation (3)

$$HG_p(s) = K_p \left[1 - e^{-Ts}\right] \frac{e^{-a\tau s}}{s(\tau s + 1)}$$

If the sampling period is equal to the delay time of the model ($T = a\tau$), which is here defined as *fast sampling*, we apply the Z transform operation (*14*) to obtain

$$HG_p(z) = K_p(1-b)\frac{1}{z(z-b)} \tag{10}$$

where

$$b \triangleq e^{-T/\tau} \tag{11}$$

If the sampling period is greater than the delay time of the model ($T > a\tau$), which is here defined as *slow sampling*, the Z transform yields (*19*)

$$HG_p(z) = K_p(1-d)\frac{\left(z + \dfrac{d-b}{1-d}\right)}{z(z-b)} \tag{12}$$

where

$$b = e^{-T/\tau}$$
$$d = e^{a - T/\tau} \tag{13}$$

For the servo problem, we consider a step in set point of size Δr

$$r(s) = \frac{\Delta r}{s}$$

for which case

$$R(z) = \Delta r \frac{z}{z-1} \tag{14}$$

For the regulator problem, the design equation, Equation (6), requires that $G_pU(z)$ be known. Consider a step disturbance of size Δu to occur at a sampling instant and enter the system at the same point as the manipulated variable. Then

$$G_p(s)U(s) = \Delta u \; K_p \frac{e^{-a\tau s}}{s(\tau s + 1)} \tag{15}$$

For fast sampling ($T = a\tau$)

$$G_pU(z) = \Delta u \; K_p\,(1-b)\frac{1}{(z-1)(z-b)} \tag{16}$$

For slow sampling ($T > a\tau$)

$$G_pU(z) = u\,K_p\,(1-d)\frac{\left(z + \dfrac{d-b}{1-d}\right)}{(z-1)(z-b)} \tag{17}$$

where b and d are defined as in Equation (13).

In this paper, the analysis and design will be restricted to $T \geqq a\tau$. This is not a restriction in concept. For a more frequently sampled system, the required digital compensation may be obtained by application of these same techniques. The work presented here suggests that there is little incentive for more frequent sampling, although no examination has been made of processes with long model delay times ($a > 1$).

SPECIFICATION OF DESIRED RESPONSE

Servo Response ($T = a\tau$)

Consider a step in set point of size Δr at time $t = 0$ in a fast sampling system ($T = a\tau$). The delay time prevents any changes in $m(t)$ from affecting the output until time $t > a\tau$. Hence, physical considerations require

$$c(t=0) = c(t=T) = 0$$

The remainder of the response at sampling instants may be arbitrarily specified.

The Z transform of the desired output may be written as

$$C_d(z) = \Delta r\,[\eta_0 + \eta_1 z^{-1} + \eta_2 z^{-2} + \ldots] \tag{18}$$

The coefficients $\{\eta_i\}$ in this series correspond to the output of the system at instants of sampling; that is

$$c(iT) = \Delta r\;\eta_i\,;\; i = 0, 1, 2, \ldots \tag{19}$$

Therefore, we must have $\eta_0 = \eta_1 = 0$; the values of η_i for $i \geqq 2$ have not yet been determined. The minimal prototype response requires $\eta_i = 1$ for $i \geqq 2$. This would result in a settling time $t_s = 2T$. Settling time in the context of this paper will include the response of the system at sampling instants only, and intersample ripple will be treated as a separate phenomenon.

The minimal prototype response does not always satisfy the application requirements previously listed for a digital compensation system and this may necessitate $t_s > 2T$ (but still finite). This implies $\eta_2 \neq 1$ [that is, $c(2T) \neq \Delta r$] in Equation (19). Possibly some of the other values of η_i might not be unity either, but if the response is to have a finite settling time, $\eta_i = 1$ for $i \geqq k$ where k is some finite positive integer. Methods for selecting η_i in these situations are discussed in reference *18*.

Servo Response ($T > a\tau$)

Again considering the servo problem, for slow sampling ($T > a\tau$) only $\eta_0 = 0$ is necessary to satisfy the physical requirements. Hence, in the minimal prototype, slow sampling, set-point response $\eta_i = 1$ for $i \geqq 1$, and $t_s = T$. The earlier remarks about meeting application requirements again apply.

Regulator Response ($T = a\tau$)

In the set-point design, by the very nature of the problem, the input can only begin to affect the manipulated variable and therefore the loop at an instant of sampling. However load disturbances can enter the loop at any time and at any point in the loop. The detailed designs presented in this paper consider only step changes in load entering the loop at the same point as does the manipulated variable and at an instant of sampling. (However, the technique is easily extended to any other situation.) For this design, a step disturbance occurring between sampling periods may not have a finite settling time. The designer could consider the more general problem to ensure finite settling for all step disturbances, but analog simulations presented below indicate that the system usually will have approximately the same response characteristics regardless of when the disturbance enters the loop, primarily because of the modeling error which is necessarily introduced. This suggests that design on the basis of disturbances occurring at only one instant during the sampling period should be sufficient.

Consider a step load of size Δu to enter a fast sampling system at an instant of sampling. Physical considerations require the value of the output to be zero for the first two instants of sampling.

$$c(t=0) = c(t=T) = 0$$

Hence, regulating feedback control is not initiated until $t = 2T$, and the output of the process will not be affected by this regulating action until $t = 3T$. Physical considerations therefore dictate the response of the system for $t = 2T$ and $t = 3T$. For the first order with delay model

$$c(2T) = \Delta u\ K_p(1 - e^{-a})$$

and

$$c(3T) = \Delta u\ K_p(1 - e^{-2a})$$

The remainder of the response can be arbitrarily specified. Hence

$$c_d(z) = \Delta u\ K_p\ [\beta_0 + \beta_1 z^{-1} + \beta_2 z^{-2} + \ldots] \quad (20)$$

where $\beta_0 = \beta_1 = 0$, $\beta_2 = 1 - e^{-a}$, $\beta_3 = 1 - e^{-2a}$, and β_i for $i \geqq 4$ are unspecified. Thus, the minimal prototype response requires $\beta_i = 0$ for $i \geqq 4$. This results in a settling time $t_s = 2T$, where the settling time for load disturbances is measured from the first sampling instant for which a nonzero error is recorded, to the sampling instant for which the output returns to the set point and maintains zero steady state error. Settling time is defined in this manner because in real systems the time or location in the loop where the disturbance occurs is unknown. Once again, no restriction has been placed on the intersample behavior in the definition of settling time. To satisfy the application constraints, $t_s > 2T$ may again be necessary.

Regulator Response ($T > a\tau$)

For slow sampling, physical considerations require

$$c(t = 0) = 0$$

and

$$c(t = T) = \Delta u\ K_p(1 - e^{a - T/\tau})$$

Hence in Equation (20) $\beta_0 = 0$, $\beta_1 = 1 - e^{a-T/\tau}$, and β_i for $i \geqq 2$ are unspecified. For a minimal prototype response, $\beta_i = 0$ for $i \geqq 2$, and the resulting settling time is $t_s = T$. Once again the application constraints may require a longer settling time.

DIGITAL COMPENSATION ALGORITHMS

The minimal prototype, digital compensation algorithms for a step in set point or load, with fast or slow sampling, are now easily derived by substituting the appropriate transforms from among Equations (10), (12), (14), (16), (17), (18), and (20) into the appropriate design equation, Equation (5) or (6). The resulting algorithms are presented in this section. Pertinent information is also included on the characteristics of the various closed-loop responses. Details on these derivations are found in reference *18*. For the cases in which the minimal prototype compensation is unsatisfactory for application to real systems, alternative nonminimal algorithms are presented which do satisfy all application requirements. This is accomplished by appropriate modification of $C_d(z)$.

The compensation algorithms $D(z)$ are presented as Z transforms in the form of a ratio of two polynomials relating the manipulated variable $M(z)$ to error $E(z)$ at sampling instants.

$$\frac{M(z)}{E(z)} = D(z) = \frac{g_0 + g_1 z^{-1} + g_2 z^{-2} + \ldots + g_k z^{-k}}{1 + h_1 z^{-1} + h_2 z^{-2} + \ldots + h_p z^{-p}} \quad (21)$$

The equivalent difference equation, suitable for digital computer programming, can be found by cross-manipulation of Equation (21), which yields

$$M(z) = [g_0 + g_1 z^{-1} + g_2 z^{-2} + \ldots + g_k z^{-k}]E(z) - [h_1 z^{-1} + h_2 z^{-2} + \ldots + h_p z^{-p}]\ M(z)$$

This relationship may be inverted by the definition of the Z transform to yield Equation (2).

The design algorithms are:

1. Fast sampling, set-point, minimal prototype design ($T = a\tau$)

$$D(z) = \frac{1}{K_p(1-b)}\ \frac{z(z-b)}{(z+1)(z-1)} \quad (22)$$

The set-point response has $t_s = 2T$ with no hidden oscillations. The load response has $t_s = \infty$ and is a monotonically decaying response.

2. Slow sampling, set-point, minimal prototype design ($T > a\tau$)

$$D(z) = \frac{1}{K_p(1-d)}\ \frac{z(z-b)}{(z-1)\left[z + \dfrac{d-b}{1-d}\right]} \quad (23)$$

The set-point response has $t_s = T$, but the response has intersample ripple with decay ratio given by $DR = (d - b/1 - d)^2$. Specification of a desired decay ratio therefore uniquely determines the required sampling period. The load response has $t_s = \infty$ and is a monotonically decaying response.

3. Slow sampling, set-point, nonminimal design ($T > a\tau$)

$$D(z) = \frac{1}{K_p(1-b)}\ \frac{z(z-b)}{(z-1)\left[z + \dfrac{d-b}{1-b}\right]} \quad (24)$$

The set-point response has $t_s = 2T$ with no hidden oscillations. The load response has $t_s = \infty$ and is a monotonically decaying response. It will be shown below that the sampling period should be chosen approximately as in the minimal prototype design (item 2 above).

4. Fast sampling, load, minimal prototype design ($T = a\tau$)

$$D(z) = \frac{(1+b+b^2)}{K_p(1-b)}\ \frac{z\left[z - \dfrac{b(1+b)}{1+b+b^2}\right]}{(z-1)[z + (1+b)]} \quad (25)$$

The algorithm is open-loop unstable. The load response has $t_s = 2T$ with no intersample ripple. The set-point response has $t_s = 3T$ with no hidden oscillations but a large overshoot.

5. Fast sampling, load, nonminimal design ($T = a\tau$)

$$D(z) = \frac{z\left[\dfrac{3}{4}\left(1 + \dfrac{2}{3}b + b^2\right) z^2 + \dfrac{1}{4}(1+b)(1-b)^2 z - \dfrac{1}{4} b(1+b)^2\right]}{K_p(1-b)(z-1)\left[z + \dfrac{1}{2}(1+b)\right]^2} \quad (26)$$

The open-loop algorithm is now stable. The load response has $t_s = 3T$ with no intersample ripple. The set-point response has $t_s = 4T$ with no hidden oscillations and less overshoot than the minimal prototype response.

6. Slow sampling, load, minimal prototype design ($T > a\tau$)

$$D(z) = \frac{(1-bd)}{K_p(1-d)^2}\ \frac{z\left[z - \dfrac{b(1-d)}{(1-bd)}\right]}{(z-1)\left[z + \dfrac{d-b}{1-d}\right]} \quad (27)$$

The load response has $t_s = T$, but the response has intersample ripple with $DR = (d - b/1 - d)^2$. The set-point response has $t_s = \infty$ and is oscillatory with $DR \rightarrow$

$(d-b/1-d)^2$. The sampling period is chosen to give the desired decay ratio.

7. Slow sampling, load, nonminimal design ($T > a\tau$)

$$D(z) = \frac{(1+b)}{K_p\,(1-d)} \frac{z\left[z - \dfrac{b}{1+b}\right]}{(z-1)\left[z + \dfrac{d-b}{1-d}\right]} \tag{28}$$

The load response has $t_s = 2T$ with no hidden oscillations. The set-point response has $t_s = 2T$, but the response has intersample ripple with $DR = (d-b/1-d)^2$. The sampling period is chosen to give the desired decay ratio.

In three of the slow sampling algorithms, $z = -(d-b)/(1-d)$ is a pole-zero cancellation location in the z domain, and the decay ratio for a step in set point is

$$DR = \left(\frac{d-b}{1-d}\right)^2 \tag{29}$$

One immediate result is that closed-loop stability requires

$$\frac{d-b}{1-d} < 1 \tag{30}$$

which implies

$$T/\tau > \ln\left[2e^a - 1\right] \tag{31}$$

Hence, specifying a desired location for this pole-zero cancellation or, equivalently, specifying a desired set-point decay ratio, uniquely determines the normalized sampling period.

$$T/\tau = \ln\left\{\frac{e^a\,(\sqrt{DR}+1)-1}{\sqrt{DR}}\right\} \tag{32}$$

For a ¼ decay ratio

$$T/\tau = \ln\left\{3e^a - 2\right\} \tag{33}$$

For slow sampling Equation (33) appears to be a good compromise between response speed and ensurance of a stable pole-zero cancellation. It is probably advisable not to sample more often than Equation (33) indicates when applying a slow sampling algorithm, although of course one may sample less often.

For the slow sampling, set-point, nonminimal design, both set-point and load responses are theoretically nonoscillatory. Unlike the other slow sampling, digital-compensation algorithms, Equation (24) is stable for all sampling periods $T/\tau \geqq a$, since the pole $z = -(d-b)/(1-b)$ always lies within the unit circle. For $T/\tau = a$, Equation (24) reduces to Equation (22). Although there are no stability constraints on the sampling period of Equation (24), experience suggests again the use of Equation (33) to determine the sampling period.

EXPERIMENTAL VERIFICATION

In evaluating any proposed control algorithm, the closed-loop system must perform satisfactorily in the presence of modeling error and noise which inevitably occur in a real system. The digital compensation algorithms of Equations (22), (24), (26), and (28), which meet all the application requirements presented previously, were tested experimentally on a two-tank water heating process to verify satisfactory performance on a real system. The ratio of the volumes of the two stirred tanks was 0.535, with the smaller tank placed downstream from the larger. The tanks were connected by a delay tube whose volume was 0.087, the volume of tank 1. The objective was to control the temperature of the water leaving the system (that is, the tank 2 water temperature) by manipulating the amount of heat input into the upstream tank, tank 1. If 5% mixing delay for each of the two tanks is included in the total delay time (*7, 13*) and if the process is assumed to act as a lumped-parameter system with no consideration to the capacitance of the containers nor to heat losses to the surroundings, the process dynamics may be theoretically described by the following transfer function

$$G_p(s) = K_p \frac{e^{-0.164\tau s}}{(\tau s+1)(0.535\tau s+1)} \tag{34}$$

The dynamic lags of the associated equipment, used to measure temperature and to manipulate heat input, and the various side capacitances and heat losses contribute additional time constants which are small with respect to the time constants of the process (*13*). For a constant flow rate of 2 liters/min., the residence time of the larger tank, tank 1, was $\tau = 2.51$ min. Hence, there are significant, realistic differences between the process and a model containing only one time constant and a delay time. Aiken (*1*) gives equipment details.

In practice a digital computer would be used for DDC. However, for research and development purposes, if a digital computer is not available, an analog computer may be relatively easily adapted for single-loop DDC studies, using "bucket brigades" of sample-and-hold amplifiers to store required past values (*18*).

The process reaction curve obtained from an open-loop step test (*26*) was modeled in the time domain to a transfer function of first order with delay by means of a direct two-parameter search. Fitting of the first 75% of the response by a least-squares criterion resulted in the model

$$G_p(s) = K_p \frac{e^{-1.2s}}{3.7s+1}; \quad a = 0.32 \tag{35}$$

Fitting of the first 99% resulted in

$$G_p(s) = K_p \frac{e^{-1.3s}}{3.6s+1}; \quad a = 0.36 \tag{36}$$

where all time constants are in minutes and $K_p = 1.15$ v./v. An arithmetic average of the two models was used as a basis for calculating the constants in the various digital compensation algorithms ($\hat{\tau} = 3.65$ min., $\hat{a} = 0.34$).

The system was then operated closed loop in the manner suggested by the block diagram of Figure 1*a*. The slow and fast sampling, set-point and load, digital compensation algorithms previously presented, Equations (22), (24), (26), and (28), were individually applied, evaluated, and where appropriate, compared with the response of a conventional controller. Each system was first evaluated for a 0.3-kw. step in the upstream tank load heater, occurring at a sampling instant. If no regulating action were taken, this disturbance would result in a net change in the output temperature of $c_u = 3.85$°F., where c_u is the uncontrolled final value for a step in load. After the output returned to its original value and steady state was again achieved, a step in set point of 1.5°F. was introduced, except in Figures 2*a* and 3*a*, *b*, and *c*, where the step size was 3°F. Numerical values for the various digital compensation algorithms may be found in reference *18*.

Load Compensation Response

Figure 2 demonstrates the transient response of the system under the load compensation algorithms and for comparison also shows a response when the system is under conventional three-mode control. In each response, the temperature of the downstream tank (that is, the con-

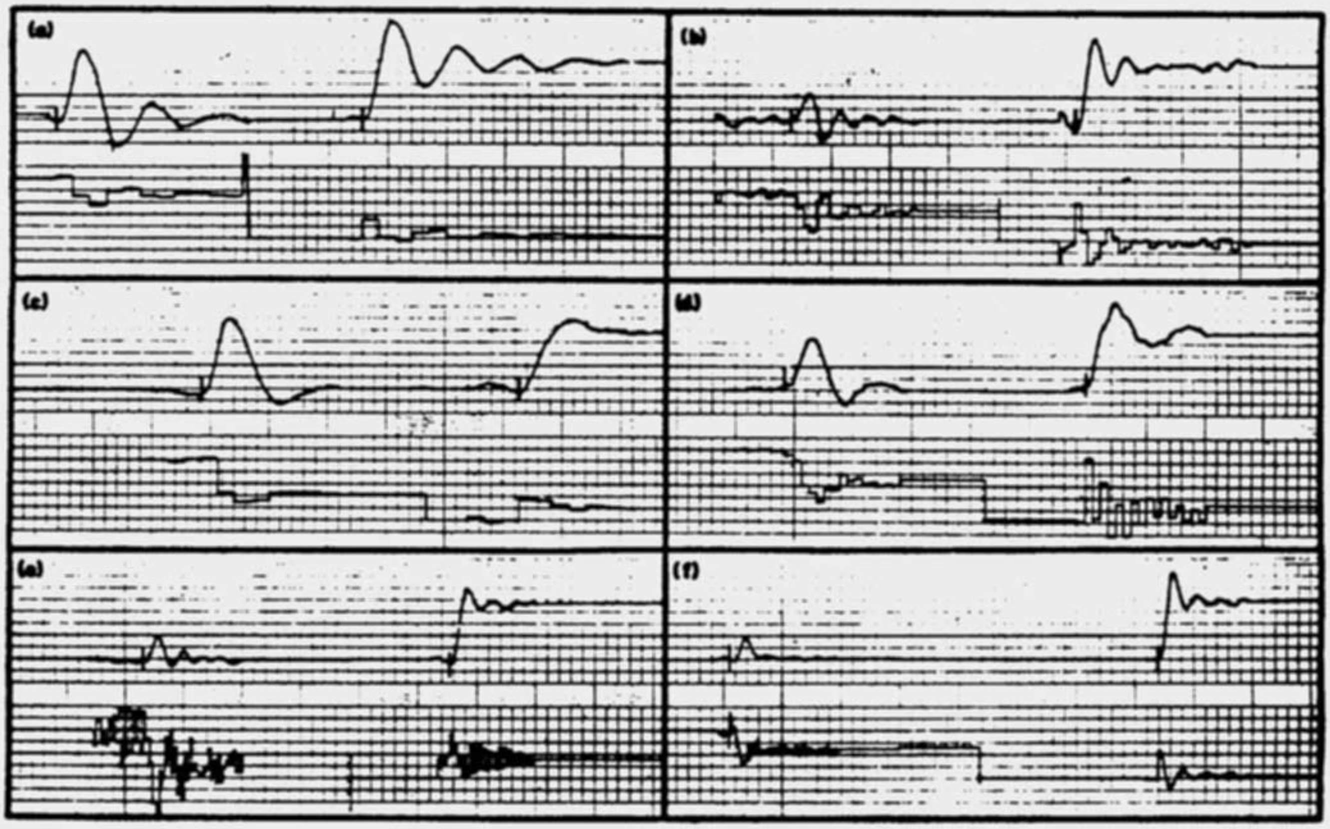

Fig. 2. Experimental load compensation responses for: (a) Slow sampling, nonminimal compensation. (b) Slow sampling, nonminimal compensation with P-D transmitter. (c) Slow sampling, nonminimal compensation with P-D transmitter [same sampling period as in (a)]. (d) Fast sampling, nonminimal compensation. (e) Fast sampling nonminimal compensation with P-D transmitter. (f) Three-mode, conventional control tuned as regulator.

trolled variable) is shown in the upper channel. The manipulated variable is shown in the lower channel. Load responses are shown to the left and set-point responses to the right in each figure.

Figure 2*a* shows the response of the system under slow sampling, load, nonminimal compensation, Equation (28). The sampling period was determined by Equation (33), $T/\hat{\tau} = 0.795$. Hence, $T = 2.9$ min.

For processes which generate only a sampled-data output (for example, chromatograph), digital compensation may be applied directly to the output of the sampling device with a sampling period equal to or slower than the operational time of the apparatus. However, for most systems continuous output information is available. Hence, the response characteristics of the system may be improved by applying a proportional-derivative (P-D) transmitter to the process output, ahead of the sampling action of the computer. The output of the transmitter then becomes the controlled variable. One such transmitter is produced by the Taylor Instrument Company (*27*) and has available derivative action from 2 to 200 sec.

If a P-D transmitter is applied to the process output and the derivative time is chosen to cancel the process minor time constant ($\tau_D = 1.34$ min.), the model of the new process can be written from Equation (34) as

$$G_p'(s) = K_p \frac{e^{-0.411s}}{2.51s + 1}; \quad \hat{a}' = 0.164 \qquad (37)$$

If Equation (33) is again used to choose the sampling period, the results are $T/\hat{\tau}' = 0.428$ and $T = 1.075$ min. The response of this system is shown in Figure 2*b* and is a considerable improvement over Figure 2*a*. Both the period and peak error in the load response, for which the system is designed, have been reduced, at the expense of some additional oscillation. Although the P-D signal is now the variable compensated for by the control algorithm, good control is also obtained on the process output as is observed from the upper channel of Figure 2*b*. It is also possible to effect this pole cancellation by providing the P-D action in the digital computer, ahead of the $D(z)$ compensation.

It is of interest to examine the system when the P-D transmitter is applied, but the same sampling period as that of Figure 2*a* is used, $T = 2.9$ min., which implies $T/\hat{\tau} = 1.156$. This is acceptable since a more stable theoretical pole-zero cancellation results. The designed decay ratio for a step in set point is theoretically reduced from 0.25 to 0.0073, and the theoretical pole-zero concellation now takes place at $z = -0.855$ instead of $z = -0.5$. However, this improvement (reduction in oscillation) is more than nullified by the reduction in performance for a step in load as is observed by comparing Figure 2*c* with Figure 2*b*.

Figure 2*d* illustrates the response of the system when the fast sampling, load, nonminimal compensation algorithm of Equation (26) is used, $T = 1.24$ min., $T/\hat{\tau} = 0.34$. This response may be compared with the output-sampled, P-I response of an earlier paper (*20*). Both systems have the same sampling period. The P-I load response has a normalized peak value, $c_{\max}/c_u = 0.50$, whereas the digital-compensation load response has $c_{\max}/c_u = 0.36$. The corresponding load decay ratios are 0.19 vs. 0.14 and the corresponding response periods are 10 min. vs. 11.2 min. Hence, the more sophisticated digital

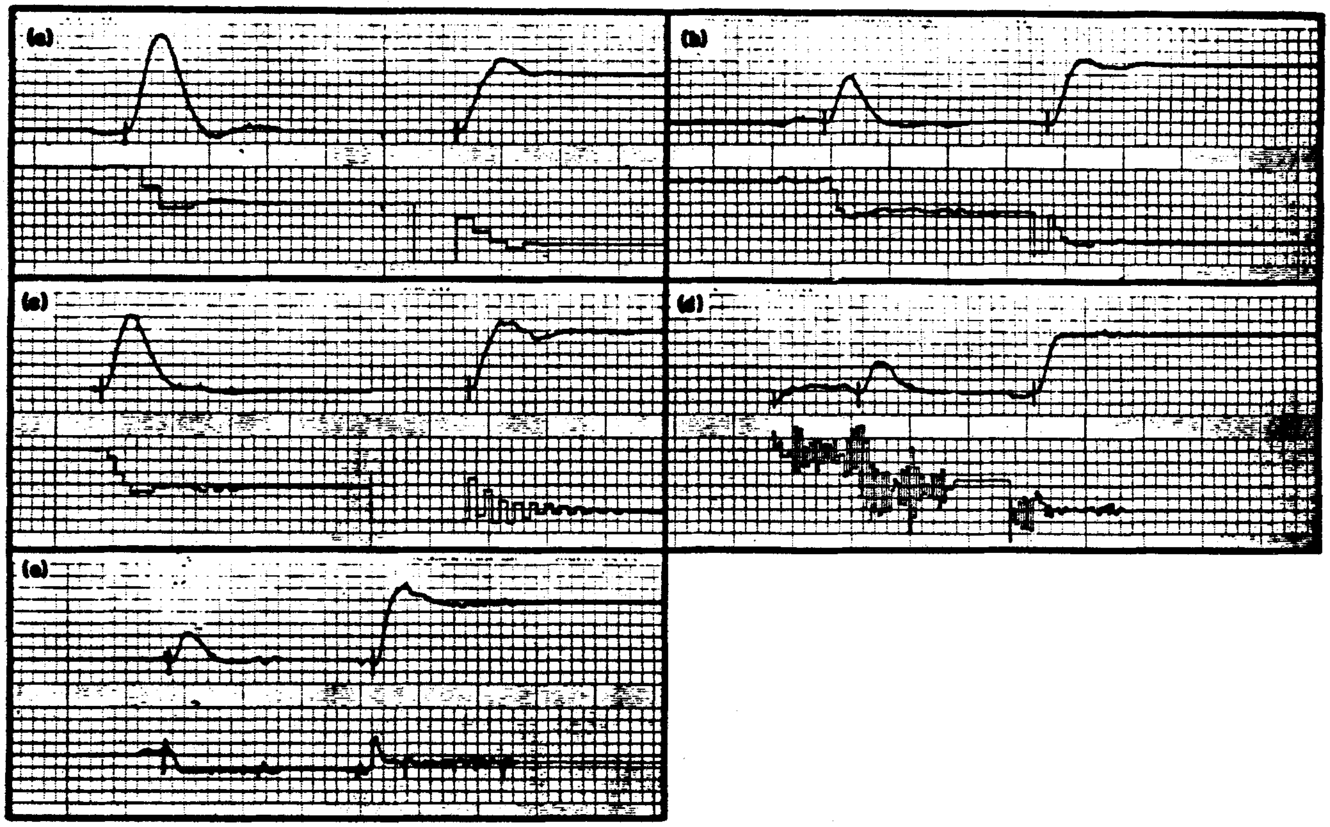

Fig. 3. Experimental set-point compensation responses for: *(a)* Slow sampling, nonminimal compensation. *(b)* Slow sampling, nonminimal compensation with P-D transmitter. (c) Fast sampling, minimal prototype compensation. *(d)* Fast sampling, minimal prototype compensation with P-D transmitter. *(e)* Three-mode, conventional control tuned as servomechanism.

compensation has generally better response characteristics than the output-sampled P-I response when both receive the same amount of information about the process output. A similar comparison of the set-point responses yields the same conclusion. In addition the fast sampling, set-point, minimal prototype response presented below compared even more favorably with the output-sampled P-I response.

Figure 2*e* shows the response when the fast sampling, load, nonminimal compensation algorithm of Equation (26) is used with the P-D transmitter on the process output ($\tau_D = 1.34$ min.). Hence, the process model is again Equation (37), $T = 0.411$ min. and $T/\hat{\tau} = 0.164$. Figure 2*e* illustrates that the loop is tightly tuned. As a result measurement noise tends to prevent the manipulated variable and the state of the system from coming to rest completely. In an effort to prevent noise from contaminating the error signal input to the digital compensator, 1-sec. filters were placed on the process output and P-D transmitter signals. This reduced the noise level to that in Figure 2*e*, and since the process time constant is so long, undoubtedly further reduction could be accomplished with higher filter time constants.

Figure 2*f* illustrates the response of the system under three-mode conventional control, optimally tuned for a step in load as suggested by Latour (*12*). The digital compensation response of Figure 2*e* compares favorably with Figure 2*f*.

Set-Point Compensation Response

Figure 3 illustrates the response of the system under the various set-point compensation algorithms, and for comparison a response when the system is under conventional, three-mode control. The information in Figure 3 is reported in the same manner as in Figure 2. The response characteristics are tabulated below.

Figures 3*a* and 3*b* illustrate responses under the slow sampling, set-point, nonminimal algorithm of Equation (24), without and with the P-D transmitter canceling the minor time constant. The same sampling periods of Figures 2*a* and 2*b* were again used.

Figures 3*c* and 3*d* illustrate responses under the fast sampling, set-point, minimal prototype algorithm of Equation (22) without and with the P-D transmitter canceling the minor time constant. The sampling periods of Figures 2*d* and 2*e* were used.

Figure 3*e* illustrates the response of the system under conventional, three-mode control, optimally tuned for a step in set point as suggested by Haalman (*5*). The compensation response of Figure 3*d* compares favorably with that of Figure 3*e*.

Analog Simulation of Water Process

Figure 4 illustrates several analog simulation transient responses with Equation (34) used to represent the dynamics of the process. Information is reported in the same manner as in Figure 2.

Figures 4*a* and 4*b* illustrate the response of the simulated system under the fast sampling, load, nonminimal compensation algorithm of Equation (26), without and with the P-D transmitter canceling the minor time constant of the process. The system has been subjected to load disturbances at $t = nT$, $(n + 1/4)T$, $(n + 1/2)T$, $(n + 3/4)T$, and finally to a step in set point. The characteristics of the response to a load disturbance are remarkably similar regardless of when the disturbance enters the system. This same phenomenon occurred when the other digital compensation algorithms were simulated. Hence

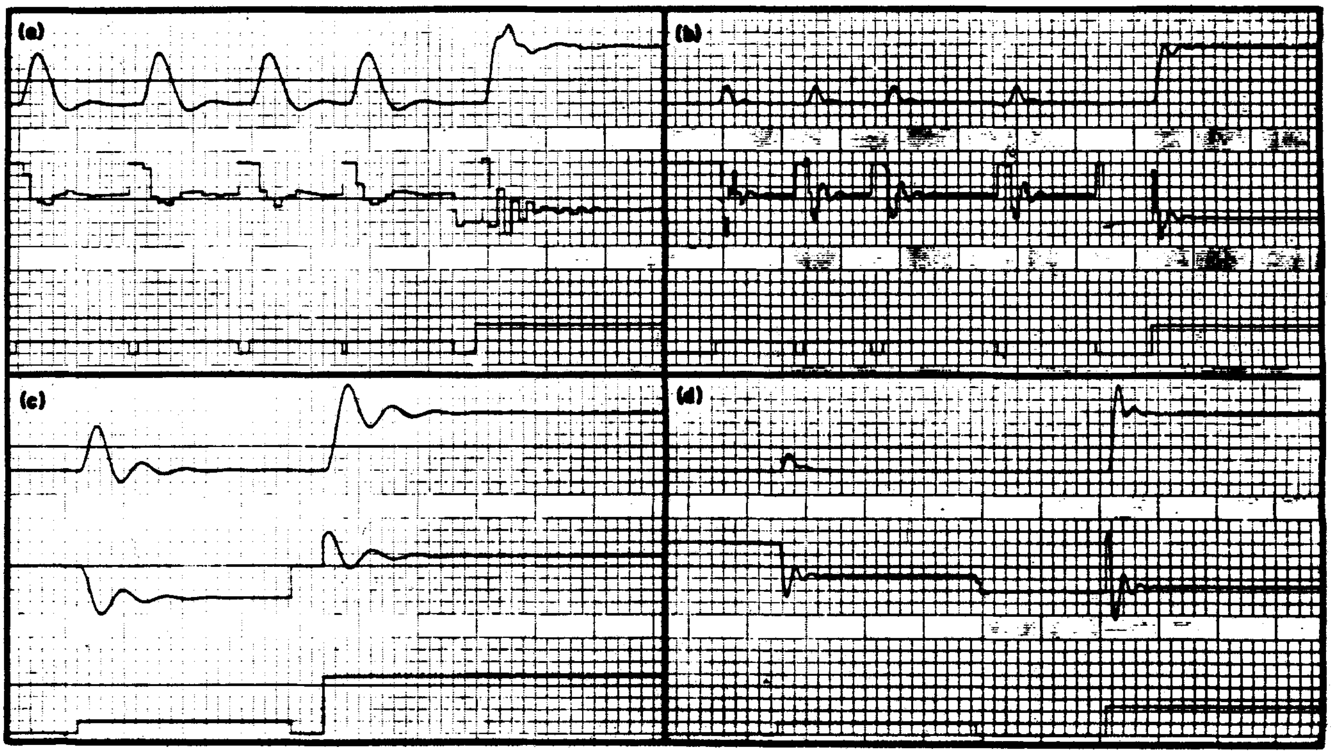

Fig. 4. Analog simulation responses for: (*a*) Fast sampling, load, nonminimal compensation. (*b*) Fast sampling, load, nonminimal compensation with P-D transmitter. (*c*) P-I conventional control. (*d*) Three-mode, conventional control tuned as regulator.

this result verifies our earlier assertion that load compensation algorithms need only be designed for loads occurring at a single instant during the sampling period.

Figure 4*c* illustrates responses for proportional integral, continuous control. The controller was tuned with the settings of Cohen and Coon (*3*). The resulting responses have decay ratios of 0.17 and 0.25, respectively, for steps in load and set point. The P-I settings of Ziegler and Nichols (*26*) resulted in a somewhat more oscillatory system.

Figure 4*d* illustrates the response of the simulated system under conventional, three-mode control with the settings of Latour (*12*) used.

The characteristics of each of the responses in Figures 2 and 3 are tabulated in Table 1 along with their design characteristics and the response characteristics of an analog simulation of the system. Included in this table are pertinent data on the response of the system under continuous, P-I-D control tuned as suggested by Latour (*12*) and Haalman (*5*). Although the settings of Latour are optimal for step disturbances in load, they have been observed to be quite sensitive to modeling error (*12*). Industrial regulator loops will more likely be tuned by the method of Cohen and Coon (*3*) or Ziegler and Nichols (*26*). Response characteristics of analog simulations of these latter systems are given in Table 2. Also included are response characteristics from analog simulations of several continuous-data loops under P-I control.

The response characteristics of the simulated system under digital compensation compare quite satisfactorily with the simulated P-I control system. For the water process this represents performance from a controller sampling once every 1.24 to 2.9 min. comparable to that from a continuous controller. The simulated digital compensation system with the P-D transmitter used yielded responses comparable to those obtainable from conventional, three-mode control. For the water process this implies a sampling period of 0.41 to 1.08 min.

Table 1 shows the experimental transient responses to be uniformly more sluggish (longer period) than their analog simulations. A reasonable explanation for this characteristic is that the dynamics of the associated equipment were not included in the analog model. This would include the noise filters, the final control element, the resistance bulb thermometer, the capacitances in the tank walls and heaters, etc. In addition, the power controller contained a small amount of hysteresis which adds phase lag to the system. The other characteristics, such as decay ratio, set-point overshoot, and c_{max}/c_u for load disturbances, all compare favorably.

Modeling error prevents the water process and analog simulation from having finite settling time and no intersample ripple. Nevertheless, the number of sampling periods required to return the system to zero error was estimated, although the system still oscillated after this number of sampling periods. These values compare favorably with the theoretical finite settling time t_s of the design.

CONCLUSIONS

Satisfactory performance of the proposed algorithms has been demonstrated when applied to a real system with higher order dynamics. The resulting model error prevents the real system from having a finite settling time. However, finite settling time is only an artificial criterion. A quantitative examination of the digital compensation transient responses shows them to compare favorably with the response of the same process under an optimally tuned, two- or three-mode, conventional controller. This latter comparison is a more realistic performance criterion.

Digital compensation algorithms have been presented only for sampling periods equal to or greater than the model delay time. Comparable algorithms can be derived for more frequent sampling, and theoretically the only

TABLE 1. CLOSED-LOOP RESPONSE CHARACTERISTICS OF TWO-TANK WATER HEATING SYSTEM UNDER VARIOUS LOAD AND SET-POINT, DIGITAL COMPENSATION ALGORITHMS

	Figure	2a		2b		2c		2d		2e		2f	
	Type of compensation	S,L,N		S,L,N,PD		S,L,N,PD		F,L,N		F,L,N,PD		PID Latour	
Desired response characteristics	Type of disturbance (L = load step) (SP = set-point step)	L	SP	L	SP	L	SP	L	SP	L	SP	L	SP
	T, min	2.9	2.9	1.08	1.08	2.9	2.9	1.24	1.24	0.411	0.411	0	0
	DR	0	0.25	0	0.25	0	0.0073	0	0	0	0	*	*
	Overshoot for set point; c_{max}/c_u for load	0.55	1.18	0.35	1.48	0.68	0.43	0.49	0.53	0.28	0.72	*	*
Experimental results	DR	0.24	0.38	0.32	0.28	0.03	0.00	0.14	0.30	0.37	0.08	0.27	0.24
	Overshoot for set point; c_{max}/c_u for load	0.47	0.75	0.26	0.46	0.48	0.21	0.36	0.56	0.15	0.25	0.15	0.52
	Period, min.	12.0	11.2	5.2	5.2	**	**	11.2	12.4	4.4	4.4	3.2	4.0
Analog simulation results	DR	0.14	0.14	0.20	0.36	0.00	0.00	0.05	0.15	0.19	0.07	0.29	0.25
	Overshoot for set point; c_{max}/c_u for load	0.47	0.92	0.17	0.25	0.49	0.14	0.35	0.38	0.12	0.25	0.11	0.50
	Period, min.	8.8	8.4	3.6	3.6	**	**	9.6	8.8	2.8	2.8	2.8	2.8

	Figure	3a		3b		3c		3d		3e	
	Type of compensation	S,SP,N		S,SP,N,PD		F,SP,M		F,SP,M,PD		PID Haalman	
	Type of disturbance (L = load step) (SP = set-point step)	L	SP	L	SP	L	SP	L	SP	L	SP
Desired response characteristics	T, min.	2.9	2.9	1.08	1.08	1.24	1.24	0.411	0.411	0	0
	Overshoot for set point; c_{max}/c_u for load	0.55	0	0.35	0	0.49	0	0.28	0	*	*
Experimental results	Overshoot for set point; c_{max}/c_u for load	0.65	0.27	0.30	0.08	0.50	0.19	0.20	0.00	0.18	0.14
Analog simulation results	Overshoot for set point; c_{max}/c_u for load	0.53	0.05	0.28	0.00	0.41	0.08	0.18	0.00	0.16	0.22

S = slow sampling; F = fast sampling; L = load design; SP = set-point design; N = nonminimal; M = minimal prototype; PD = used P-D transmitter on output; PID = used conventional three-mode controller; * = no value specified by this design; ** = no obtainable value. See appropriate figure.

TABLE 2. RESPONSE CHARACTERISTICS OF SIMULATED CONTINUOUS-DATA SYSTEMS [PROCESS GIVEN BY EQUATION (34)]

Type of control	P-I Cohen-Coon		P-I Ziegler-Nichols		P-I Haalman		P-I-D Cohen-Coon		P-I-D Ziegler-Nichols	
Type of disturbance (L = load step) (SP = set-point step)	L	SP	L	SP	L	SP	L	SP	L	SP
DR	0.17	0.25	0.36	0.46	0.09	0.10	0.08	0.06	0.13	0.13
Overshoot for set point; c_{max}/c_u for load	0.31	0.50	0.25	0.63	0.36	0.21	0.21	0.42	0.16	0.63
Period, min.	7.6	7.6	5.8	5.8	8.8	8.8	6.4	6.2	5.2	5.2
Controller settings										
K_{loop}	2.73		4.55		1.96		4.17		6.06	
τ_I, min.	2.44		3.22		3.65		2.69		1.93	
τ_D, min.	0		0		0		0.43		0.48	

requirement is that more values of the manipulated variable and error must be stored. However, our experimental work indicates that algorithms which require more frequent sampling cause the system to become more sensitive to noise and modeling error. Lindorff (*14*) reports that sensitivity to a change in gain increases exponentially with the order of the poles at the origin. For processes containing delay time, more frequent sampling results in more poles at the origin (*19*). Kalman and Bertram (*10*) suggest that the sampling period be not less than 10% of the dominant time constants of the plant. In Figures 2*e* and 3*d*, the sampling period is 16% of the major time constant of the process, and the transient responses indicate that little is to be gained by sampling more frequently, which reinforces the rule of Kalman and Bertram.

Designs have been presented only for a first order with delay model. An examination of higher order models containing delay may be made with identical techniques. The latter algorithms will require more information storage. However, the more accurate description of the dynamics of the actual process will result in more of the model dynamics being included in the minimum-phase part of the model. Thus, the pure delay element in a higher order model will be correspondingly smaller than the delay time in a first-order with delay model, and this would result in the ability to sample somewhat more frequently without theoretically increasing the order of the characteristic equation. However, the results obtained in the present work when the P-D transmitter was used suggest that noise problems may be encountered.

The DDC users (*25*) have arbitrarily suggested the following sampling rates for DDC:

Flow process	1/sec.
Pressure and level processes	1/5 sec.
Temperature processes	1/20 sec.

The present work demonstrates that the sampling frequency may at times be decreased by an order of magnitude over these values and still yield a system with comparable response characteristics. The selection of a sampling period should be based on the model dynamics of the process, rather than on the type of output to be controlled, using systematic design procedures such as presented here. This large allowable decrease in sampling frequency should significantly reduce the cost of the input-output peripheral equipment associated with any DDC system. The present cost of this associated equipment is a considerable portion of the overall expenditure.

ACKNOWLEDGMENT

The financial assistance of The Esso Research and Engineering Company, The Monsanto Company, The Procter and Gamble Company, and the generous donation of computing time by the Purdue University Computer Science Center are gratefully acknowledged.

NOTATION

a = process delay time as a fraction of the major time constant
b = $\exp\{-T/\tau\}$
$C(z)$ = Z transform of system output
$C_d(z)$ = Z transform of desired system output
$c(t)$ = output of the system
$c_d(t)$ = desired output of the system
c_c = sampled (or clamped) output
c_{max} = maximum value of output when system is forced by a load disturbance
c_u = final value output would reach if no regulating action were taken by controller when system is forced by a load disturbance
$D(z)$ = Z transform of digital compensation algorithm
d = $\exp\{a - T/\tau\}$
$e(t)$ = error of the system
$G_c(s)$ = controller transfer function
$G_p(s)$ = process transfer function
$H(s)$ = zero-order hold transfer function
K_p = process gain
$m(t)$ = value of final control element in the system
$m(nT)$ = piecewise constant final control element which results for example when digital compensation is used to control the system
$R(s)$ = set-point transfer function
$r(t)$ = set point
T = sampling period
T/τ = normalized sampling period
t_s = theoretical finite settling time
$U(s)$ = load transfer function
$u(t)$ = load variable
Δr = size of step in set point
Δu = size of step in load
τ = process major time constant
τ_D = controller or transmitter derivative time constant
τ_I = controller integral time constant

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Manuscript received May 23, 1966; revision received November 4, 1966; paper accepted November 7, 1966. Paper presented at AIChE Houston meeting.